

Characterizing subsurface gas through Bayesian inversion of a seismic attenuation model

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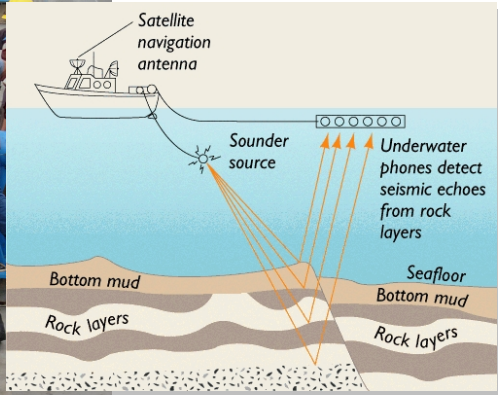
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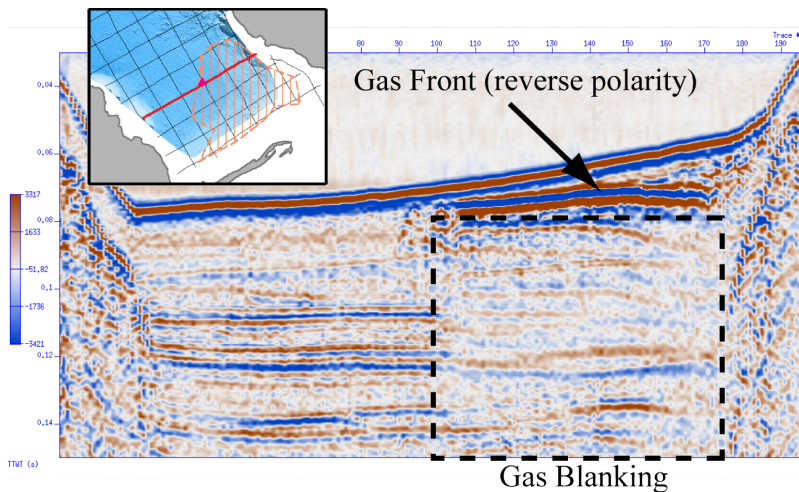
Motivation



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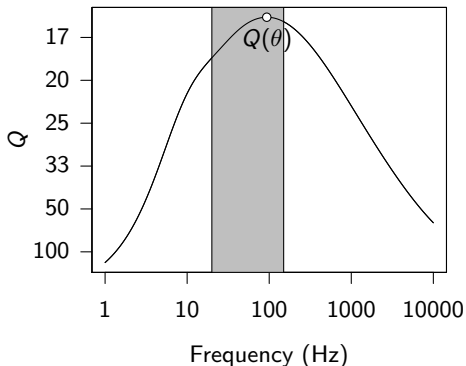
Attenuation of seismic reflection signals



In: Morgan et al. (2010).

The P-wave attenuation model of Carcione and Picotti (2006)

13 parameters in set θ define the attenuation curve (left):

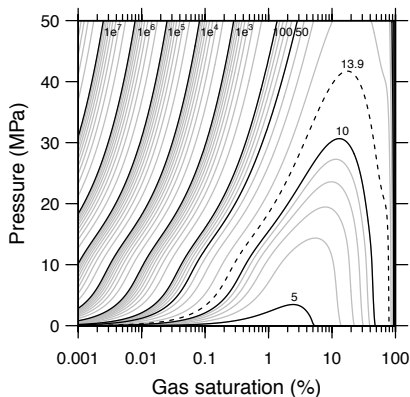
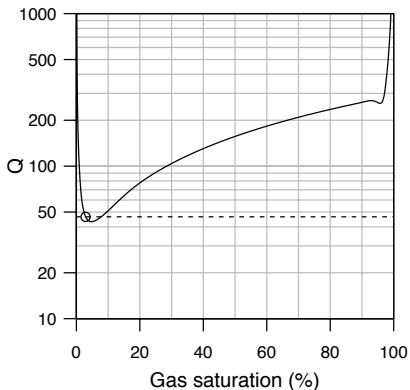


- 1 ϕ : soil porosity
- 2 κ : soil permeability
- 3 K_s : solid grain bulk moduli
- 4 μ_s : solid grain shear moduli
- 5 ρ_s : solid grain density
- 6 K_w : water bulk moduli
- 7 ρ_w : water density
- 8 η_w : viscosity of water
- 9 d : total layer thickness
- 10 S_g : gas saturation
- 11 η_g : viscosity of gas
- 12 P : total pressure
- 13 T : temperature

I pick $Q(\theta)$ as the minimum Q of the curve within the range of seismic frequencies.

Model behavior

Attenuation model is nonlinear and multimodal



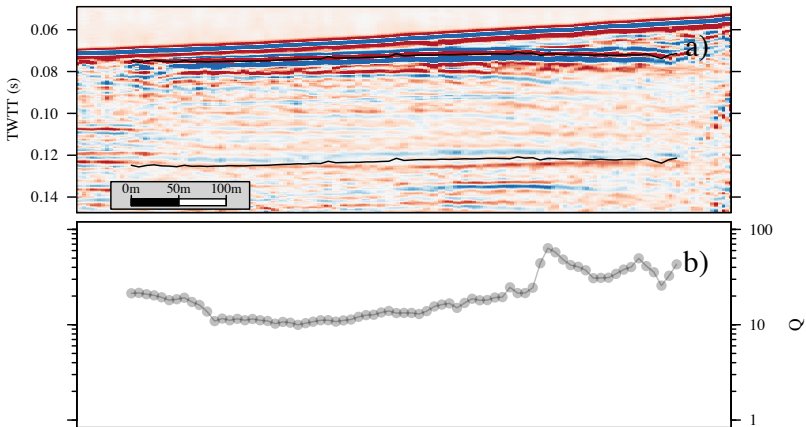
Method overview

Our goal is to link the theoretical attenuation model to observations of attenuation from seismic data. This is a two step process:

- 1 Measure quality factor (Q) from seismic data
- 2 Invert model: find set of parameters (θ) that match modeled $Q(\theta)$ to measured \hat{Q} . Here, I explore two ways to do this:
 - 1 Genetic algorithm (Morgan et al., 2012)
 - 2 Bayesian hierarchical model (Morgan et al., 2014)

I validate the genetic algorithm method at the well-studied Blake Ridge (offshore the Carolinas) where free gas lies below gas hydrate. I test both inversion methods at Finneidfjord, Norway, where gas exists at a much shallower depth and in smaller concentration.

Finneidfjord



Bayesian Hierarchical Model

With $\hat{Q} = (\hat{Q}_1, \dots, \hat{Q}_m)^T$ measured at m locations (traces) with unknown parameter values $\theta = (\theta_1, \dots, \theta_m)^T$, where each θ_i is a vector containing values for each of the 13 parameters,

$$\hat{Q} = Q(\theta) + e.$$

$Q(\theta)$ is the attenuation forward model (“simulator”) and e contains both measurement error from \hat{Q} and model error from $Q(\theta)$. Then, our joint posterior is:

$$p(\theta | \hat{Q}) \propto L(\hat{Q} | Q(\theta)) \times \pi(\theta),$$

with likelihood coming from assuming e is Gaussian with zero mean:

$$L(\hat{Q} | Q(\theta)) \propto \exp\left(-\frac{1}{2\sigma^2}(\hat{Q} - Q(\theta))^T(\hat{Q} - Q(\theta))\right),$$

where σ is the standard deviation of \hat{Q} .

The prior contains the spatial smoothness constraint^a that our random variables behave as a Markov random field over space:

$$\pi(\theta) \propto \exp \left(\beta \sum_{i \sim j} u(\theta_i - \theta_j) \right),$$

where the sum is over all sets of nearest neighbors ($i \sim j$) and

$$u(d) = \begin{cases} \frac{1}{s}(1 - (d/s)^3)^3, & \text{if } -s < d < s \\ 0, & \text{if } |d| \geq s \end{cases}$$

is the tricube function. Pragmatically, $\beta = 0.5$ and $s = 0.3$ work well. The distance between vectors ($d = \theta_i - \theta_j$) is found via Mahalanobis distance with the covariance matrix Σ_θ set *a priori*.

^aHigdon, D., Reese, C.S., Moulton, J.D., Vrugt, J.A., and Fox, C. 2008. Posterior exploration for computationally intensive forward models. In: Handbook of Markov Chain Monte Carlo. Boca Raton, Florida: CRC Press.

MCMC: Single-site Metropolis Algorithm

Pseudo-code:

initialize θ

for $k = 1 : niter$ **do**

for $i = 1 : m$ **do**

$\theta'_i = \theta_i + z$, where $z \sim MVTN(0, \Sigma_\theta, a, b)$

if $u < \frac{p(\theta'_i | \hat{Q})}{p(\theta_i | \hat{Q})}$, where $u \sim U(0, 1)$ **then**

$\theta_i \leftarrow \theta'_i$

end if

end for

end for

At Finneidfjord we have $m = 83$ locations, set $niter = 100,000$, use burn-in of 10,000, and keep every 10^{th} θ_i after that. We get an acceptance rate of 49%.

Sample Space

Initial guesses (θ_1) and bounds taken from literature or geometry of seismic data:

	Global (both sites)					
	Lower bound	Initial value	Upper bound			
Gas saturation S_g (%)	0	1	100			
Porosity ϕ	0.38 ^a	0.55	0.73 ^a			
Permeability κ (Darcy)	10^{-8} ^a	10^0	10^5 ^a			
Solid grain bulk modulus K_s (GPa)	20 ^b	30	70 ^b			
Solid grain shear modulus μ_s (GPa)	5 ^b	13	50 ^b			
Solid grain density ρ_s (g/cc)	2.55 ^b	2.65	2.71 ^b			
Water bulk modulus K_w (GPa)	2.00	2.25 ^c	2.50			
Water density ρ_w (g/cc)	1.000	1.025	1.030			
Water viscosity η_w (Pa·s)	0.001	0.003 ^c	0.005			
Gas viscosity η_g (10^{-4} Pa·s)	1.0	1.5 ^c	2.0			
	Blake Ridge			Finneidfjord		
	Lower bound	Initial value	Upper bound	Lower bound	Initial value	Upper bound
Total pressure P (MPa)	~32.41	~32.93	~35.19	~0.55	~0.77	~2.93
Temperature T ($^{\circ}$ C)	11	12	16	0	5	10
Layer thickness d (m)	70	75	80	36.5	37.5	38.5

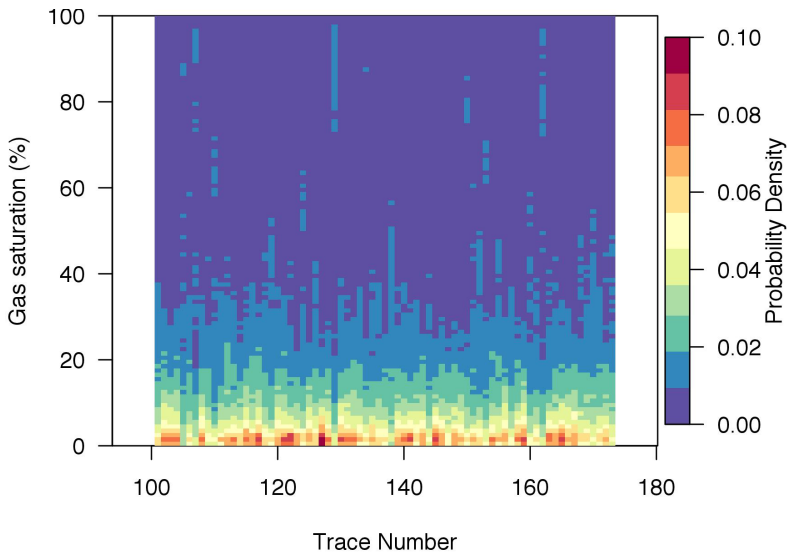
^a Schön (1996)

^b Mavko et al. (1998)

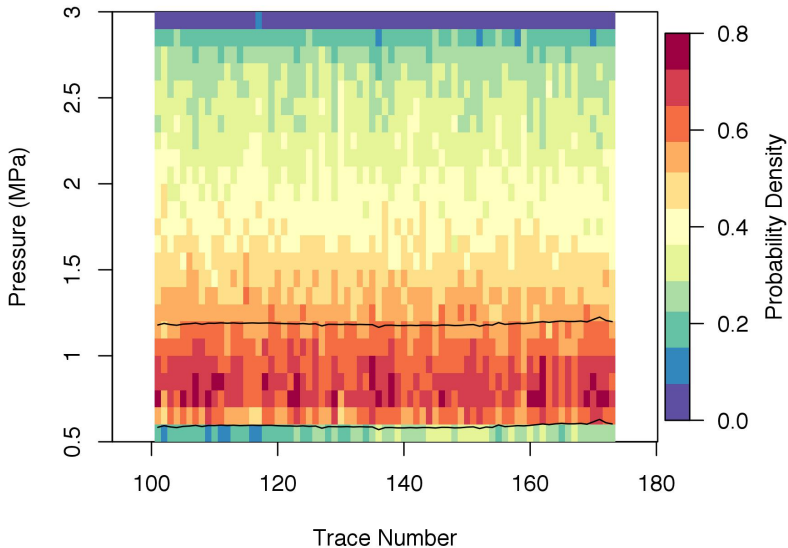
^c Carcione and Picotti (2006)

Bounds represent likely or possible conditions, and make the inversion a constrained optimization problem.

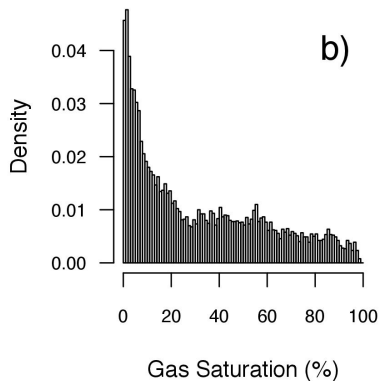
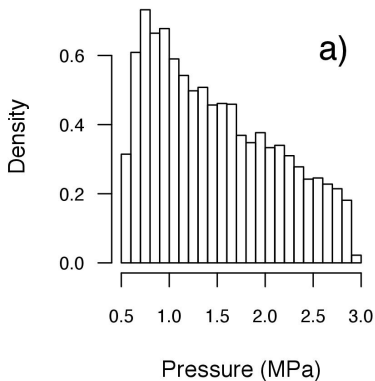
Gas saturation posterior distributions



Pressure posterior distributions



Posterior distributions: Trace 160



Ardmucknish Bay, Oban, Scotland: CO₂ release experiment

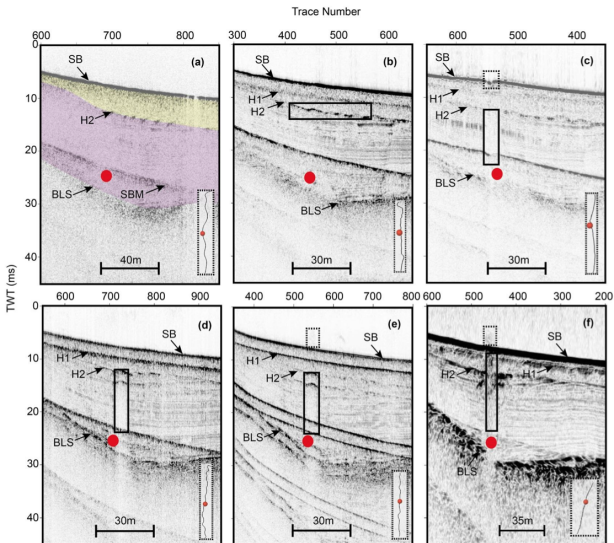
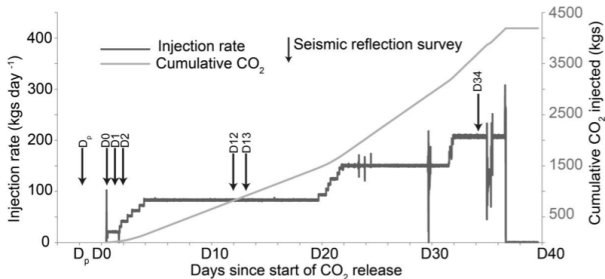


Figure from Cevatoglu et al. (*in review*)

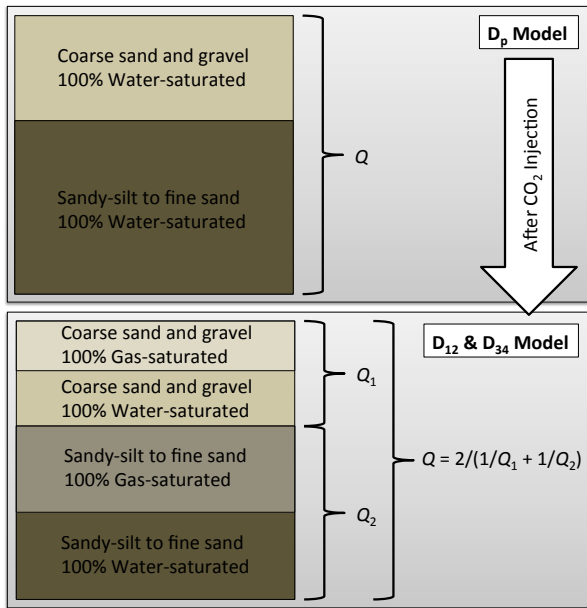
Bayesian Updating for CO₂ Injection



Goal: Estimate gas storage over time.

- Use Bayesian Updating with seismic attenuation:
 - D_p: feed in naive priors, get posteriors (without gas saturation)
 - D₁₂: feed in posteriors from D_p as priors, get new posteriors (with gas saturation)
 - Repeat over future surveys

Stratigraphic Model



Bayesian Updating scheme

Begin with prior based on geologic knowledge of system at D_p with random variables θ_p :

$$\pi(\theta_p) \sim MVTN(\mu_p, \Sigma_p, a_p, b_p) \quad (1)$$

Fit model to D_p data:

$$p(\theta_p | \hat{Q}_p) \propto L(\hat{Q}_p | Q(\theta_p)) \times \pi(\theta_p) \quad (2)$$

where

$$L(\hat{Q} | Q(\theta)) \propto \exp\left(-\frac{1}{2\sigma^2}(\hat{Q} - Q(\theta))^T(\hat{Q} - Q(\theta))\right) \quad (3)$$

$$\sigma = 2 \quad (4)$$

Bayesian Updating scheme (cont'd)

Update with D12 data, using both posterior of Dp and expert knowledge of new variables (θ_{new}) to define prior:

$$p(\theta_{12}|\hat{Q}_{12}) \propto L(\hat{Q}_{12}|Q(\theta_{12})) \times \pi(\theta_{12}) \quad (5)$$

$$\pi(\theta_{12}) \sim MVTN(\boldsymbol{\mu}_{12}, \boldsymbol{\Sigma}_{12}, \mathbf{a}_{12}, \mathbf{b}_{12}) \quad (6)$$

$$\boldsymbol{\mu}_{12} = [\overline{p(\theta_p)}, \mu_{new}] \quad (7)$$

$$\boldsymbol{\Sigma}_{12} = \begin{bmatrix} \text{cov}(p(\theta_p)) & \boldsymbol{\Sigma}_{p,new} \\ \boldsymbol{\Sigma}_{new,p} & \boldsymbol{\Sigma}_{new} \end{bmatrix} \quad (8)$$

$$\mathbf{a}_{12} = [\mathbf{a}_p, \mathbf{a}_{new}] \quad (9)$$

$$\mathbf{b}_{12} = [\mathbf{b}_p, \mathbf{b}_{new}] \quad (10)$$

Bayesian Updating scheme (cont'd)

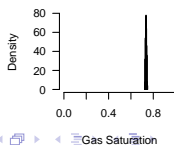
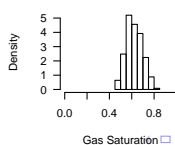
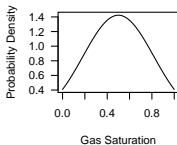
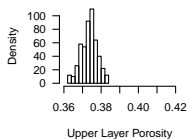
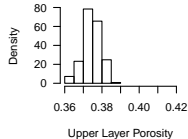
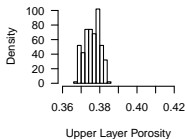
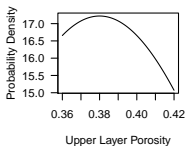
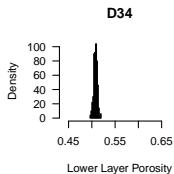
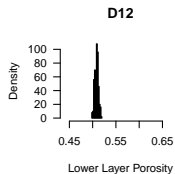
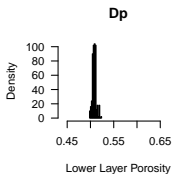
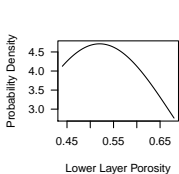
Repeat at next time step:

$$p(\theta_{34} | \hat{Q}_{34}) \propto L(\hat{Q}_{34} | Q(\theta_{34})) \times \pi(\theta_{34}) \quad (11)$$

$$\pi(\theta_{34}) \sim MVTN(\overline{p(\theta_{12})}, \text{cov}(p(\theta_{12})), a_{12}, b_{12}) \quad (12)$$

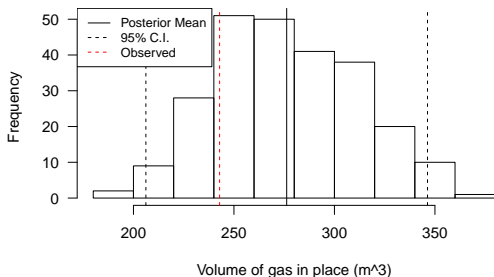
Continue with more data as it becomes available during gas injection. This process gains strength over time as priors become more informed, but is also flexible to adapt to changing conditions as they appear in the data (\hat{Q}).

Results: Prior/Posterior Comparison



Results: Volume of Gas at D12

- **Observed:** $(800 \text{ kg})(1 - 0.15 \text{ seepage rate}^a) / 2.8 \text{ kg/m}^3$ (for CO_2 at 0.15 MPa and 12 °C)
- **Posterior:** $\sum_{i=\text{layers}} \phi_i \times S_{g,i} \times h_i \times (A = 10\text{m}^2)^b$

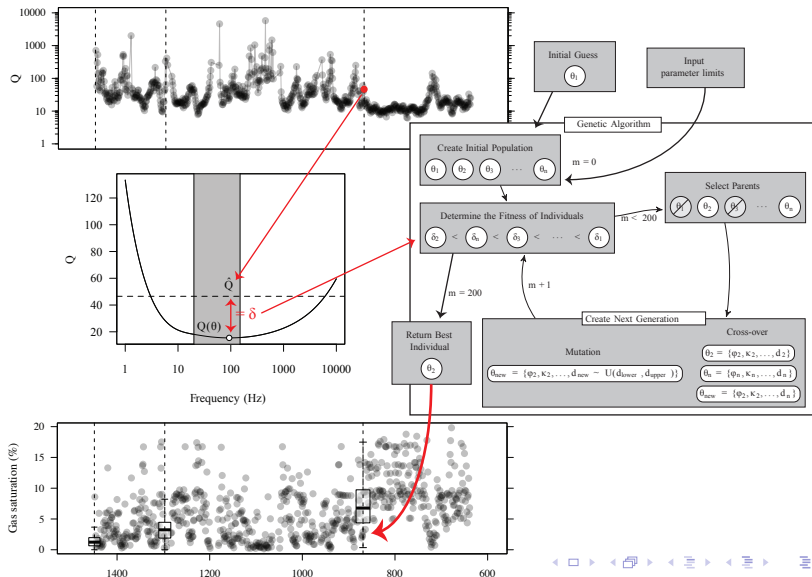


^aBlackford et al. (2014). Detection and impacts of leakage from sub-seafloor deep geological carbon dioxide storage. *Nature Climate Change*. DOI: 10.1038/NCLIMATE2381.

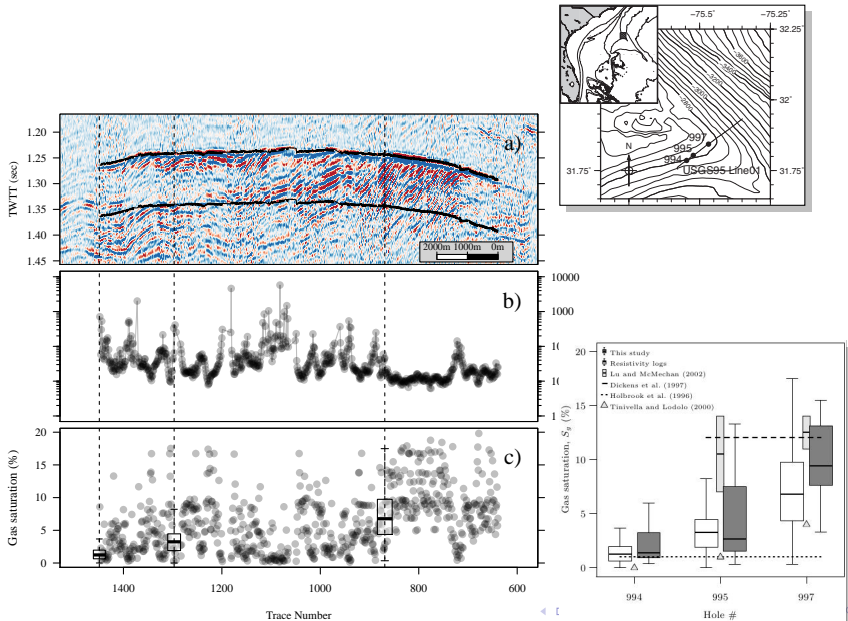
^bCevatoglu et al. (*in review*). Gas migration pathways, controlling mechanisms and changes in sediment physical properties observed in a controlled sub-seabed CO₂ release experiment.

Genetic Algorithm Inversion

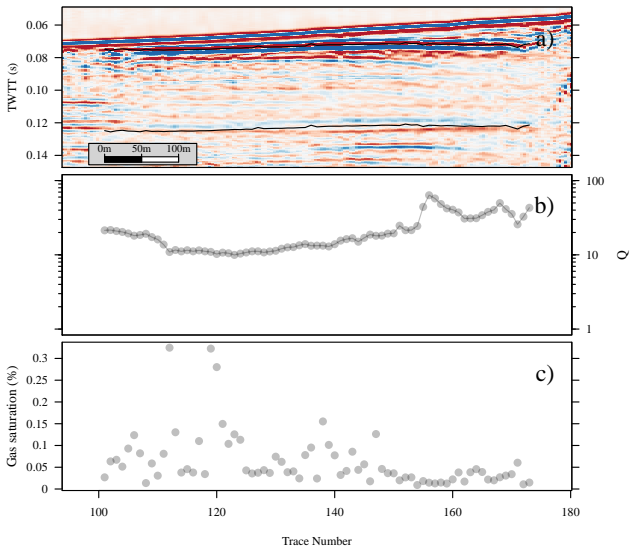
Invert attenuation model to get gas saturation estimates



Blake Ridge



Finneidfjord



Conclusions

Seismic data can offer first-order estimates of gas properties over wide areas.

- Here, we estimate gas saturation and pressure by inverting attenuation model
 - No borehole data needed - no calibration required
 - Constrain the parameter space by realistic values from literature and seismic data
- Our estimates of S_g generally agree with borehole data at Blake Ridge, as well as between inversion methods
- Give reasonable gas volumes at Oban CO₂ injection site
- This methods work in shallower environments; are sensitive to small S_g

Conclusions

Compare how methods perform with attenuation model:

- Genetic Algorithm
 - Precise
 - Doesn't explore all modes well
 - Treats traces independently
 - Quick (can parallelize traces)
- Bayesian Hierarchical Model
 - Accounts for spatial correlation
 - Posteriors show modes
 - Slow (can't parallelize)

Bayesian Updating at single site gathers strength over time.

- More data is always better
- Priors refine over time
- Posterior uncertainty lessens over time

References

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